27/9/22 Operation Research

Defination → A method of mathematically based analysis for providing a quantative basis for management decision.

→ OR deals with development and application of advance analytical method to improve decision making. Operations research aurives at optimal Or near-Optimal Solution to complex decision making problems.

↳ using minimum resources maximum time and taking benifits.

Management Application of OR :—

→ finance ⇒ ⓐ Budgeting & investment

1) Capital, requirement, cash flow analysis
2) Credit policies credit risk.
3) Investment decision. profit plan for company
4) assests & liabilities of company
↳ cash ↳ bank depts
↳ investment ↳ mortage debt
↳ realstate ↳ ~~wages~~
↳ taxes

→ Purchasing procurement & Exploration :— Rules for buying supplies and variing the prices

1) quantities and time of purchases
2) Bidding policies
3) replacement policy.

→ Procurement - act of obtaining or purchasing goods or ser typically for business purpose.

Exploration.

→ **Production management** :-
- physical distribution ~~of~~ as location size and warehouses
- facilated planning → no. of factories, Warehouses Hospitals etc.
- Manufacturing → product ~~manag~~ maintanance, & scheduling
- maintanance & project scheduling
  → resource allocation
  → Marketing
  → no. of salesman
  → time of marketing

→ Personal management
- mix of age + skills (experience)
- Research & Development
- Cost

24/09/22

Main phases of OR :-

1. Formulating the problem
2. Constructing a mathematical model (Actual situation or Analysis)
3. Deriving the solution from the model
4. Testing the model & its solution - (Updating the model)
5. Controlling the solution
6. Implementing the Solution

To formulate the problem we need :-

- Objective function (max or min form)
- constraints or Assumptions.
  ↳ these are in variable form. limitations or conditions.

→ Decision making also depends on model.

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Model :-

1. Classification by Structure
   ↳ Iconic model → used for Scaling (up/down)
   ↳ Analog model → one set of property used to represent another set of property
   ↳ Symbolic model (mathematical model) → Sign language.

2. Classification by purpose :-
   ↳ Discriptive model → It is based on some situation, like servey, question hour, Observation result
   ↳ Predictive model → Based on some prediction conditions
   ↳ Prescriptive model → Based on both its prescribed condition

3. classification by nature of environment or system :-
   ↳ Determinatic model → Based on assumption, assume the co[ndition] by perfect knowledge.
   ↳ Probablistic model → yes/No basis.

(4) classification by behaviour:-

↳ Static models → Independent on time

↳ Dynamic model → Changes based on time, Dependent.

(5) classification by method of solution:-

↳ Analytical model → Based on Analysis

↳ Simulation models.

(6) classification by use of digital computers:-

↳ Analog and mathematical models combined

↳ function models.

↳ Quantitative models

○ Heuristic models.

⇒ Slack $\le$ (+)

Surplus $\ge$ (−)

Slack	$\le$ (+)	Slack
Surplus	$\ge$ (−)	Surplus

Slack.	$\le$ (+)
Surplus	$\ge$ (−)

12/10/22

Simplex method :-

Q.) Consider LPP maximum $Z = 3x_1 + 2x_2$ subject to constrain (restriction)

$x_1 + x_2 \le 4$ — add slack variable $\le$ +

$x_1 - x_2 \le 2$ — subtract surplus variable $\ge$ −

and $x_1, x_2 \ge 0$

Solution) Express the problem in standard form by introducing slack or surplus variable to convert the inequality constraints into equation.

$x_1 + x_2 + S_1 = 4$

$x_1 - x_2 + S_2 = 2$

∴ S_1 and S_2 are slack variable with cost Zero

Simplex Method :-

for converting General form into Standard form

(1) Check whether the ~~LPP~~ objective function of LPP is maximized or minimized. If it is to be minimized then we convert it into a problem of maximization by

$$\text{Max } Z = -\text{Min } Z$$

(2) Check all the decision variable are ≥ 0 if any dv are unrestricted

$2x_1 + x_2 \le 4$, $x_1 \ge 0$, x_2 unrestricted

$2x_1 + (x_2' - x_2'') \le 4$

$x_1, x_2', x_2'' \ge 0$

Slack variable $\leq$ add

Surplus variable $\geq$ subtract

By youtube

Q.) Max $z = 3x_1 + 2x_2 + 5x_3$

$x_1 + 2x_2 + x_3 \leq 430$

$3x_1 + 2x_3 \leq 460$

$x_1 + 4x_2 \leq 420$

$x_1, x_2, x_3 \geq 0$

Sol) By introducing slack variables s_1, s_2, s_3, convert the problem in standard form.

$$\text{Max } z = 3x_1 + 2x_2 + 5x_3 + 0s_1 + 0s_2 + 0s_3$$

Subject to, $x_1 + 2x_2 + x_3 + s_1 = 430$

$3x_1 + 2x_3 + s_2 = 460$

$x_1 + 4x_2 + s_3 = 420$

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

An initial basic feasible solution is given by

$x_1 = x_2 = x_3 = 0$, $s_1 = 430$; $s_2 = 460$ $s_3 = 420$

(It was given so it's 0)

Writing in matrix form, $AX = B$

$$\begin{array}{cccccc} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 \end{array}$$
$$\begin{bmatrix} 1 & 2 & 1 & 1 & 0 & 0 \\ 3 & 0 & 2 & 0 & 1 & 0 \\ 1 & 4 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} 430 \\ 460 \\ 420 \end{bmatrix}$$

Initial simplex table. → By objective function coefficient fill

Incoming vector

		C_j	3	2	5	0	0	0	Minimum ratio
C_B	B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	X_B/x_3
0	s_1	430	1	2	1	1	0	0	430/1 = 430
0	s_2	460	3	0	$\boxed{2}$	0	1	0	460/2 = 230 ←
0	s_3	420	1	4	0	0	0	1	—
$Z_j - C_j = -C_j$			-3	-2	-5 ↑	0	0	0	

outgoing vector; key element; key row; key column

Cost ← taken variable; B value

By matrix

$Z_j - C_j = C_B \times (X_j) - C_j$

$Z_1 - C_1 = C_B X_1 - C_1$
$= (0 \times 1) - 3$
$= -3$

$Z_2 - C_2 = C_B X_2 - C_2$
$= 0 \times 3 - 2$
$= -2$

Minimum ratio
$= \frac{X_B}{C_K}$, $C_K > 0$
$= 430/1 = 430$.

value will be same

First iteration

Incoming

		C_j	3	2	5	0	0	0	Min
C_B	B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	X_B/x_2
0	s_1	200	-1/2	$\boxed{2}$	0	1	-1/2	0	200/2 = 100 ←
5	x_3	230	3/2	0	1	0	1/2	0	→ divided by key element
0	s_3	420	1	4	0	0	1/2	1	420/4 = 105
$Z_j - C_j$		-	7/2	-2 ↑	0	0	5/2	0	

outgoing

in place of outgoing vector X_3 is incoming vector written

Calculation for the table at back page

$= \frac{430\times2-460\times1}{2} = \frac{860-460}{2} = 200$

$= \frac{1\times2-3\times0}{2} = 2/2 = 1$

$\frac{1\times2-3\times1}{2} = \frac{2-3}{2} = -1/2$

$\frac{420\times2-460\times0}{2} = \frac{840}{2} = 420$

$\frac{2\times2-0\times1}{2} = 4/2 = 2$

$\frac{4\times2-0\times0}{2} = 8/2 = 4$

$\frac{0\times2-1\times1}{2} = -1/2$

$\frac{2\times0-0\times1}{2} = 0$

$Z_j - C_j = C_B X_i - C_j$

$Z_1 - C_1 = C_B X_1 - C_1$

$= (0\times1/2 + 5\times3/2 + 0\times1) - 3$

$= 15/2 - 3 = 9/2$

2nd iteration

		C_j	3	2	5	0	0	0
C_B	B	X_B	x_1	x_2	x_3	S_1	S_2	S_3
2	x_2	100	$-1/4$	1	0	$1/2$	$-1/4$	0
5	x_3	230	$3/2$	0	1	0	$1/2$	0
0	S_3	20	2	0	0	-2	1	1
$Z_j - C_j$			4	0	0	1	2	0

$R_3' = R_3 - 4R_1$

Since all $Z_j - C_j \geq 0$ then our solution is optimum.

$x_2 = 100, \quad x_3 = 230$

$x_1 = 0$

$Max\ z = C_B X_B$

$= (2\times100 + 5\times230 + 0\times20)$

$= 200 + 1150$

$= 1350$ Ans

Q. Consider LPP maximum $Z = 3x_1 + 2x_2$ subject to constraint

$x_1 + x_2 \leq 4$

$x_1 - x_2 \leq 2$

$x_1, x_2 \geq 0.$

Sol:- $Max\ Z = 3x_1 + 2x_2 + 0S_1 + 0S_2$

subject to $x_1 + x_2 + S_1 = 4$

$x_1 - x_2 + S_2 = 2$

$x_1, x_2, S_1, S_2 \geq 0$

initial feasible solution is given by

$x_1 = x_2 = 0, \quad S_1 = 4 \quad S_2 = 2$

Matrix

$$\begin{array}{cccc} x_1 & x_2 & s_1 & s_2 \end{array}$$
$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

Initial table simplex

		Cj	3	2	0	0	min ration X_B/X_1
CB	B	XB	X_1	X_2	S_1	S_2	
0	S_1	4	1	1	1	0	$4/1 = 4$
0	S_2	2	$\boxed{1}$	-1	0	1	$2/1 = 2$ ←
$Z_j - C_j$ =			-3	-2	0	0	

$Z_j - C_j = C_B x_j - C_j$

$Z_1 - C_1 = C_B X_1 - C_1$

$= (0\times1 + 0\times1) - 3$

$= -3$

		Cj	3	2	0	0
CB	XB	B	x_1	x_2	s_1	s_2
0	s_1	4	1	1	1	0
3	x_1	2	1	-1	0	1

$R_1 = R_1 - R_2$

		Cj	3	2	0	0	min R
CB	XB	B	x_1	x_2	s_1	s_2	
0	s_1	2	0	$\boxed{2}$	1	-2	1 ←
3	x_1	2	1	-1	0	1	-2 ×
$Z_j - C_j$			0	-5	0	3	

↑ entering

$(0+3) - 3$
$\Rightarrow 0$

$R_1 = R_1/2$

		Cj	3	2	0	0
CB	XB	B	x_1	x_2	s_1	s_2
2	x_2	1	0	1	1/2	-1/2
3	x_1	2	1	-1	0	1

$R_2 \Rightarrow R_2 + R_1$

CB	XB	B	x_1	x_2	s_1	s_2
2	x_2	1	0	1	1/2	-1/2
3	x_1	3	1	0	1/2	1/2
$Z_j - C_j$			0	0	+ve	+ve

$x_2 = 1 \quad x_1 = 3$

$\max Z = 3x_1 + 2x_2$
$= 9 + 2 = 11$

Q→ Min $Z = x_1 - 3x_2 + 2x_3$

subject to,

$3x_1 - x_2 + 2x_3 \le 7$

$-2x_1 + 4x_2 \le 12$

$-4x_1 + 3x_2 + 8x_3 \le 10$

and $x_1, x_2, x_3 \ge 0$

Sol⇒ By introducing slack variables s_1, s_2, s_3 convert the problem in standard form.

For converting it into max we multiply (−1)

$$\text{Max } Z' = -x_1 + 3x_2 - 2x_3 + 0s_1 + 0s_2 + 0s_3$$

Subject to $3x_1 - x_2 + 2x_3 + s_1 = 7$

$-2x_1 + 4x_2 + s_2 = 12$

$-4x_1 + 3x_2 + 8x_3 + s_3 = 10$

⇒ An initial basic feasible solution

$x_1 = x_2 = x_3 = 0 \;;\; s_1 = 7,\; s_2 = 12,\; s_3 = 10$

Writing matrix from $AX = B$

$$\begin{array}{cccccc} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 \end{array}$$
$$\begin{bmatrix} 3 & -1 & 2 & 1 & 0 & 0 \\ -2 & 4 & 0 & 0 & 1 & 0 \\ -4 & 3 & 8 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 12 \\ 10 \end{bmatrix}$$

An initial simplex table.

		C_j	-1	3	-2	0	0	0	Min ratio
C_B	B	X_B	X_1	X_2	X_3	S_1	S_2	S_3	X_B/x_2 (key column) $x_2 > 0$
0	s_1	7	3	-1	2	1	0	0	—
0	s_2	12	-2	$\boxed{4}$	0	0	1	0	$12/4 = 3$ ←
0	s_3	10	-4	3	8	0	0	1	$10/3 = 3.33$
	$Z_j - C_j$		1	-2 ↑	2	0	0	6	

For initial table process ↓

→ In c_j we fill the cost of main Z one expression

→ In B column we write the variables which we have introduced here they are S_1, S_2, S_3

→ C_B is cost of B

→ X_B is from matrix B value

Now we calculate $Z_j - C_j$

$Z_j - C_j = C_B X_j - C_j$

$\boxed{\text{In first table } Z_j - C_j = -C_j}$ because CB value is 0 in initial

→ Now we will get the minimum ratios first from $Z_j - C_j$ we will see the most negative one, and that corresponding column will be called key column here it is X_2

→ and X_2 is incoming vector

→ Now min ratio is found out by X_B/ key column (x_2) and it should be ≥ 0 otherwise we will not calculate

→ Now in this min the row will be key row and then that ~~row~~ S_2 will be outgoing vector

First iteration

		C_j	-1	3	-2	0	0	0	min ratio
CB	B	XB	x_1	x_2	x_3	S_1	S_2	S_3	$X_B/X_1,\ x_1 \geq 0$
0	S_1	10	$\boxed{5/2}$	0	2	1	1/4	0	$10/5/2 = \frac{20}{5} = 4$ ←
3	x_2	3	-1/2	1	0	0	1/4	0	–
0	S_3	1	-5/2	0	8	0	-3/4	1	–
	$Z_j - C_j$		-1/2 ↑	0	2	0	3/4	0	

Now this table process →

→ C_j value will be same as the start one

→ for B column now the incoming vector will come in place of S_2

→ Now in C_B we will put the ~~coeffi~~ cost of B.

→ Now in key row of first table will be divided by key element.

$\boxed{R_2' \Rightarrow R_2/4}$

$\boxed{R_1' \to R_1 + R_2'}$

$R_1 \to 7\ 3\ -1\ 2\ 1\ 0\ 0$

$R_2' \to 3\ -1/2\ 1\ 0\ 0\ 1/4\ 0$

$R_1'\ \ 10\ 5/2\ 0\ 2\ 1\ 1/4\ 0$

$\boxed{R_3' \to R_3 - 3R_2'}$

$R_3 \to 10\ -4\ 3\ 8\ 0\ 0\ 1$

$3R_2' \to 9\ -3/2\ 3\ 0\ 0\ 3/4\ 0$

$1\ -5/2\ 0\ 8\ 0\ -3/4\ 1$

Second iteration

		C_j	-1	3	-2	0	0	0
CB	B	XB	X_1	X_2	X_3	S_1	S_2	S_3
-1	x_1	4	1	0	4/5	2/5	1/10	0
3	x_2	5	0	1	2/5	2/10	3/10	0
0	S_3	11	0	0	10	1	-1/2	1
	$Z_j - C_j$		0	0	12/5	1/5	4/5	0

$\boxed{R_1' \to R_1 \times 2/5}$

$\boxed{R_2' \to R_2 + \frac{R_1'}{2}}$

$R_2 \to 3\ -1/2\ 1\ 0\ 0\ 1/4\ 0$

$\frac{R_1'}{2} \to 2\ 1/2\ 0\ 2/5\ 2/10\ \frac{1}{20}\ 0$

$R_2'\ \ 5\ 0\ 1\ 2/5\ 2/10\ 3/10\ 0$

$\boxed{R_3' \to R_3 + \frac{5R_1'}{2}}$

$R_3 \to 1\ -5/2\ 0\ 8\ 0\ -3/4\ 0$

$\frac{5}{2}R_1' \to 10\ 5/2\ 0\ 2\ 1\ 1/4\ 1$

$11\ 0\ 0\ 10\ 1\ -1/2\ 0\ 1$

Since all $Z_j - C_j \geq 0$, then solution is optimum

∴ The optimal solution is given by

$\text{Max } Z' = C_B X_B$

$= (-1 \times 4 + 3 \times 5 + 0 \times 11$

$= -4 + 15 = 11$

$\min Z = -\text{Max}(Z') = -11$

$x_1 = 4,\ x_2 = 5,\ x_3 = 0$ Ans

X_B we have taken

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Graphical Method

Any LPP (linear programming problem) can be solved by graphical method if it have two decision variables.

Q) Find a geometrical interpretation and solution as well. for the following LPP.

$\leq$ Below

Max $Z = 3x_1 + 5x_2$

Sub to constraint $x_1 + 2x_2 \leq 2000$

$x_1 + x_2 \leq 1500$

$x_2 \leq 600$

and $x_1, x_2 \geq 0$.

$\geq$ above

Sol) Replace all the inequality constraints by equation

$x_1 + 2x_2 = 2000$ — (1)

$x_1 + x_2 = 1500$ — (2)

let $x_1 = 0$ in eq (1)

$0 + 2x_2 = 2000$

$= \boxed{x_2 = 1000}$

let $x_2 = 0$ in eq (1)

$\boxed{x_1 = 2000}$

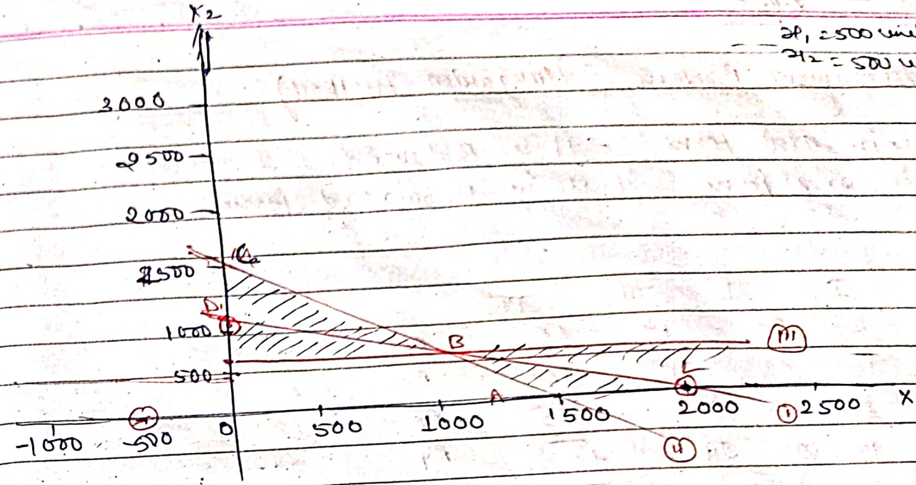
(ii) $x_1 + x_2 = 1500$

$\boxed{x_1 = 1500}$ $\boxed{x_2 = 1500}$

(iii) $\boxed{x_2 = 600}$

$x_1 = 2500$ units

$x_2 = 500$ units



Max $Z = 3x_1 + 5x_2 = 0$

$= 300x_1 + 500x_2 = 0$

$= x_1/x_2 = \frac{-500}{300}$

$\boxed{x_1 = -500 \quad x_2 = 300}$

$= A(x_1, x_2)$	$B(x_1, x_2)$	$C(x_1, x_2)$	$D(x_1, x_2)$
$= (1500, 0)$	$(980, 550)$	$(700, 600)$	$(0, 1000)$
$= 3(1500)+5(0)$ $= 4500$	$= 3(980)+5(550)$ $= 5690$	$= 2100 + 3000$ $= 5100$	$= 5000$

Q-> Max $Z = 8000x_1 + 7000x_2$

Sub to constraint $3x_1 + x_2 \leq 66$

$x_1 + x_2 \leq 45$

$x_1 < 20$

$x_2 \leq 40$

$x_1, x_2 \geq 0$.

Done

Assignment Problem (Hungarian method)

It is in $(n \times n)$ form - it is Balanced
in $(n \times m)$ form it is in unbalanced form

Q →

	I	II	III	IV	
A	8	26	17	11	R_1
B	13	28	4	26	R_2
C	38	19	18	15	R_3
D	19	26	24	10	R_4

→ Finding the minimum element in Row and subtracting it from everything in that Row.

	I	II	III	IV
A	0	-18	7	3
B	9	24	0	22
C	23	4	3	0
D	9	16	14	0
	C_1	C_2	C_3	C_4

This column

← Do the same thing for column in which it don't have 0.

	I	II	III	IV
A	$\boxed{0}$	14	7	3
B	9	20	$\boxed{0}$	22
C	23	$\boxed{0}$	3	0 ×
D	9	12	14	$\boxed{0}$

→ [assign 0 in such way that each row & column is convered].
- the 0 which is assigned the other 0 which is in same column or row will get cross.

original value ↓

A → I + B → III + C → II + D → IV

= 8 + 4 + 19 + 10 = 41 Ans

Q →

	I	II	III	IV
A	12	30	21	15
B	18	33	9	31
C	44	25	24	21
D	23	30	28	14

→ minimum in each row

	I	II	III	IV
A	0	18	9	3
B	9	24	0	22
C	23	4	3	0
D	9	16	14	0

← Now column without 0 doing the same

	I	II	III	IV
A	$\boxed{0}$	14	9	3
B	9	20	$\boxed{0}$	22
C	23	$\boxed{0}$	3	0 ×
D	9	12	14	$\boxed{0}$

→ assigning 0

A → I + B → III + C → II + D → IV → original value.

= 12 + 9 + 25 + 14 = 60 Ans

17/10/22 Q → A car hire company has one car at each of 5 depots a, b, c, d, e a customer requires a car in each town namely A, B, C, D, E. distance (in kms) between depots (origins) and towns are given in the following matrix

How should cars be assigned to customers so as to minimize the distance travelled

	a	b	c	d	e	
A	160	130	175	190	200	R_1
B	135	120	130	160	175	R_2
C	140	110	155	170	185	R_3
D	50	50	80	80	110	R_4
E	55	35	70	80	105	R_5

	a	b	c	d	e
A	30	0	45	60	70
B	15	0	10	40	55
C	30	0	45	60	75
D	0	0	30	30	60
E	20	0	35	45	70

~~Subtract~~ Find minimum no. of ~~each~~ each row and subtract it from that row

for columns do the same in which 0 don't occurs

	a	b	c	d	e	
A	30	$\boxed{0}$	35	30	15	
B	15	0	$\boxed{0}$	10	0	L_2
C	36	0	35	30	20	
D	$\boxed{0}$	0	20	0	5	L_3
E	20	0	25	15	15	

- Cross 0 horizontally and vertically covering maximum 0.

L_1

	a	b	c	d	e
A	15	0	20	15	$\boxed{0}$
B	15	15	$\boxed{0}$	10	0
C	15	$\boxed{0}$	20	15	5
D	$\boxed{0}$	15	20	0	5
	5	0	10	$\boxed{0}$	0

Now take the minimum number and subtract it from remaining and add to the intersection

Now assign 0 so that it come to row & column

→ e · B → C C → b d → a E → d original

200 + 130 + 110 + 50 + 80 = 570

Q →

	1	2	3
m_1	52	19	20
m_2	8	83	24
m_3	42	35	89

m_1	33	0	1
m_2	0	75	16
m_3	7	0	54

	1	2	3
m_1	33	0	$\boxed{0}$
m_2	$\boxed{0}$	75	15
m_3	7	$\boxed{0}$	53

= Original value

= $M_1 \to 3$

$M_2 \to 1$

$M_3 \to 2$

= 8 + 20 + 35

= 63

Ans

Done

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Transportation method:-

(i) North-west corner method (stepping stone method)

Q-> Find the basic initial feasible solution of the following transportation problem.

Warehouse / Factory	w_1	w_2	w_3	w_4	Factory Capacity
f_1	5(19)	2(30)	50	10	7/0
f_2	70	6(30)	3(40)	60	9/0
f_3	40	8	4(70)	14(20)	18/0
Requirement	5 X	8/0	7/0	14	34

-> Here checking the Requirement and factory capacity min value and according to that (5,7) 5 is minimum so will multiply 5 to 19

Now will check for row column both it should become 0 if it become 0 we will skip that here 5-5=0

$= 5\times19 + 2\times30 + 6\times30 + 3\times40 + 4\times70 + 14\times20$

$= 95 + 60 + 180 + 120 + 280 + 280$

$= 1015$

Defn -> The Transportation problem is to transport various amounts of a single homogeneous commodity that are initially stored at various origins, to different destinations in such a way that the total transportation cost is a minimum.

Done

(ii) Row Minima Method

~~(i) Row minim North-west Corner method (Stepping stone method)~~

Warehouse capacity	w_1	w_2	w_3	w_4	factory capacity
R_1	19	30	50	7(10)	7/0 X
R_2	70	8(30)	40	60	9/1
R_3	5(40)	80	6(70)	7(20)	18/11/6/0
Requirement	5	8/0	7/6/0	14/7/0	34

1st row

min (19, 30, 50, 10) = 10 checking the min value

min (7, 14) = 7 Taking the min and multiplying 14 by min value and subtracting from Requirement & factory capacity until it gets 0.

2nd row (70, 30, 40, 60) -> 30

min(8, 9) = 8

3rd row (40, 80, 70, 20) -> 20

= (7, 18) -> 7

=> $8\times30 + 1\times40 + 5\times40 + 6\times70 + 7\times20 + 7\times10$

$= 240 + 40 + 200 + 420 + 140 + 70$

$= 1110$ Ans

(iii) Lowest cost entry method :

19	36	50	7(10)	7/0
~~5~~ (70)	30	7(40)	60	9/2/0
3(40)	8(8)	70	7(20)	18/10
5/2/0	8/0	7/0	14/7/0	

minimum in the whole table = 8

min(10)

min(20) min(19) → completed because it has 0 in ~~any~~ row or column any one

min(30) ×

min(40)

min(50) × min(60) × min(70)

$= 7\times10 + 2\times70 + 7\times40 + 3\times40 + 8\times8 + 7\times20$

$= 70 + 140 + 280 + 120 + 64 + 140$

$= 814$ Ans

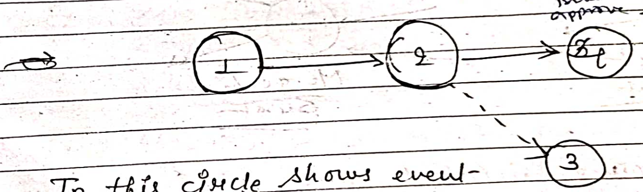
31/10/22

PERT / CPM

~~Pert~~ Project Evolution Review Technique.
This is that technique of project management which is used to manage uncertain activities of any project.

CPM → Critical path method is that technique of project management which is used to manage only certain activities of any project.

complete, start, issue, approve. ← Event Example →
design completed, pipe line laid etc.



Activities →
Preparing budget,
mixing of concrete.

In this circle shows event and arrow shows the activity.
The dotted line is dummy activity.

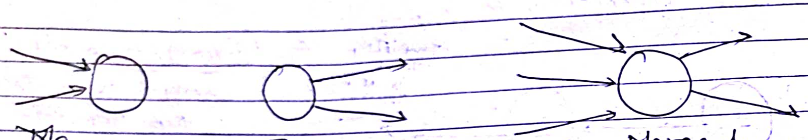
An activity which does not consume any kind resource but merely depicts the technological dependence is called a dummy activity.

Event → An event represent a point in time signifying the completion of some activities and the beginning of new ones. This is represented by circle 'O'.

i) Merge event :- When more than one activity comes and joins an event. Such event is known as merge event.

ii) **Burst event** → When more than one activity leaves an event. Such event is known as Burst event.

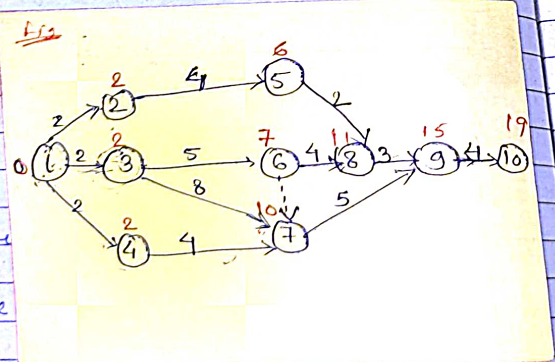
iii) **Merge & Burst event** :→ An activity may be a merge and burst event at the same time as with respect to some activities it can be a merge event and w.r.t some other activities it may be a burst event.



Merge event | Burst event | Merge & Burst

Q) Consider the following network and find out the critical path method, according to the Fulkerson's Rule.

Calculate the forward and backward pass computation.



Sol) For this, it is necessary earliest and latest each activity in the

For forward path computation

E_i = the earliest expected occurence time of event i.
L_j = the latest allowable event occurrence time of event j.
D_{ij} = the expected duration to complete the activity i-j.

Calculation
starting should be taken 0.

$E_1 = 0.$

$E_2 = \max[E_i + D_{ij}]$
$= \max[E_1 + D_{12}]$
$= 0 + 2 = 2$

Likewise value of E_3, E_4, E_5 and E_6

$E_3 = E_1 + D_{13} = 0 + 2 = 2$	$E_4 = E_1 + D_{14} = 0 + 2 = 2$	$E_5 = E_2 + D_{25} = 2 + 4 = 6$	$E_6 = E_3 + D_{36} = 2 + 5 = 7$

Consider node 7 there are three emerging activities i.e E_7

$E_7 = \max[E_3 + D_{37} = 2 + 8 = 10, E_4 + D_{47} = 2 + 4 = 6, E_6 + D_{67} = 0 + 7 = 7]$
$= 10$

$E_8 = \max[E_5 + D_{58} = 6 + 2 = 8, E_6 + D_{68} = 7 + 4 = 11] = 11$
$E_9 = \max[E_8 + D_{89} = 11 + 3 = 14, E_7 + D_{79} = 10 + 5 = 15] = 15$

$E_{10} = E_9 + D_{9,10} = 15 + 4 = 19$

Here 1-3, 3-7, 7-9 and 9-10. This longest path is called the critical path.

Now for Backward pass computation :-

Determination of Latest Time (L_i) : Backward pass computation

$L_i = E_i$

$L_i = \min\{L_j - D_{ij}\}$

Now the calculation

$L_{10} = 19$

$L_9 = \min\{L_j - D_{ij}\}$
$= \min\{L_{10} - 4\}$
$= 19 - 4$
$= 15$

$L_8 = \min\{L_9 - D_{8,9}\}$
$= 15 - 3$
$= 12$

$L_7 = \min\{L_9 - D_{7,9}\}$
$= 15 - 5$
$= 10$

$L_6 = \min\{L_{\cancel{9}} - D_{6,\cancel{7}}\}$
$= \{L_8 - D_{6,8}\}$
$= \left\{\begin{array}{l} 10 - 0 = 10 \\ 12 - 4 = 8^* \end{array}\right\} = 8$

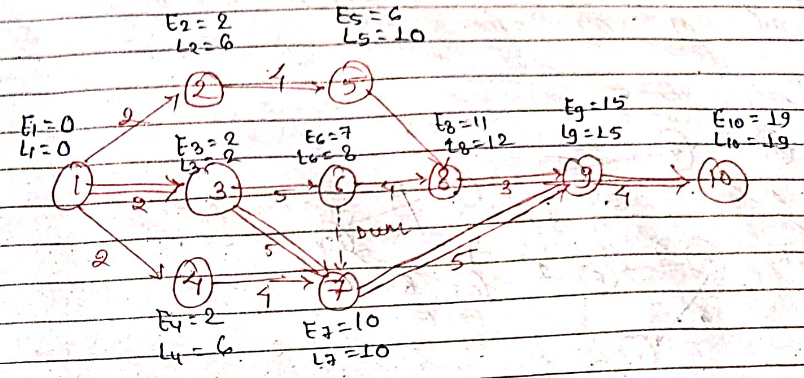
$L_5 = \min\{L_8 - D_{5,8}\}$
$= 12 - 2$
$= 10$

$L_4 = \min\{L_7 - D_{4,7}\}$
$= 10 - 4$
$= 6$

$L_3 = \min\left\{\begin{array}{l} L_6 - D_{3,6} \\ L_7 - D_{3,7} \end{array}\right\}$
$= \left\{\begin{array}{l} 8 - 5 = 3 \\ 10 - 8 = 2^* \end{array}\right\}$
$= 2$

$L_2 = \min\{L_5 - D_{2,5}\}$
$= 10 - 4$
$= 6$

$L_1 = \min\left\{\begin{array}{l} 6 - 2 = 4 \\ 2 - 2 = 0^* \\ 6 - 2 = 4 \end{array}\right\}$
$= 0$



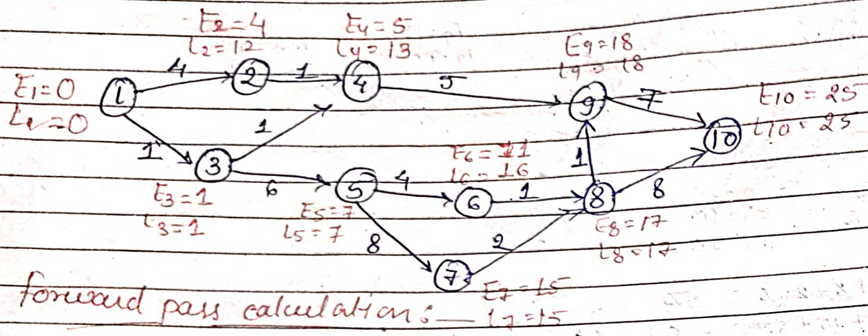
$= 19 + 15 + 12 + 8 + 2 + 0 = 56$
$= 19 + 15 + 12 + 10 + 6 + 0 = 62$
$= 19 + 15 + 10 + 6 + 0 = 50$
$= 19 + 15 + 10 + 2 + 0 = 46$

46 $10 \to 9 \to 7 \to 3 \to 1$ path

Q → A project have the time.

Activity	Weeks	Activity	Weeks
1-2	4	8-10	8
1-3	1	9-10	7
2-4	1		
3-4	1		
3-5	6		
4-9	5		
5-6	4		
5-7	8		
6-8	1		
7-8	2		
8-9	1		

Construct pert Network & calculate forward & Backward pass computation



forward pass calculation:—

$E_1 = 0$

$E_2 = \max[E_1 + D_{1,2}]$
$= 0+4 = 4$

$E_3 = \max[E_1 + D_{1,3}]$
$= 0+1$
$= 1$

$E_4 = \max\begin{Bmatrix} E_2 + D_{2,4} \\ E_3 + D_{3,4} \end{Bmatrix}$
$= \begin{matrix} 4+1 = 5^* \\ 1+1 = 2 \end{matrix} = 5$

$E_5 = \max\{E_3 + D_{3,5}\}$
$= 1+6 = 7$

$E_6 = \max\{E_5 + D_{5,6}\}$
$= 7+4 = 11$

$E_7 = \max\{E_5 + D_{5,7}\}$
$= 7+8$
$= 15$

$E_8 = \max\begin{Bmatrix} E_6 + D_{6,8} \\ E_7 + D_{7,8} \end{Bmatrix}$
$= \begin{matrix} 11+1 = 12 \\ 15+2 = 17^* \end{matrix} = 17$

$E_9 = \max\begin{Bmatrix} E_4 + D_{4,9} \\ E_8 + D_{8,9} \end{Bmatrix}$
$= \cancel{18+7 = 25}$
$= \cancel{17+8 = 25}$ $\cancel{= 25}$

$= \begin{matrix} 5+5 = 10 \\ 17+1 = 18^* \end{matrix} = 18$

$E_{10} = \max\begin{Bmatrix} E_9 + D_{9,10} \\ E_8 + D_{8,10} \end{Bmatrix}$
$= \begin{matrix} 18+7 = 25 \\ 17+8 = 25 \end{matrix} = 25$

$\begin{cases} 0+4+5+18+25 = 52 \\ 0+1+7+15+17+25 = 65 \\ 1+7+11+17+18+25 = 79 \end{cases}$

$1 \to 2 \to 4 \to 9 \to 10$

Backward pass calculation:—

$L_{10} = 25$

$L_9 = \min\{L_{10} - D_{9,10}\}$
$= 25-7$
$= 18$

$L_8 = \min\begin{Bmatrix} L_9 - D_{8,9} \\ L_{10} - D_{8,10} \end{Bmatrix}$
$= \begin{matrix} 18-1 = 17 \\ 25-8 = 17 \end{matrix} \Big\} 17$

$L_7 = \min\{L_8 - D_{7,8}\}$
$= 17-2 = 15$

$L_6 = \min\{L_8 - D_{6,8}\}$
$= 17-1$
$= 16$

$L_5 = \min\begin{Bmatrix} L_6 - D_{5,6} \\ L_7 - D_{5,7} \end{Bmatrix}$
$= 16-4$
$= 12$

$= 15-8$
$= 7^*$

$= 7$

$L_4 = \min\{L_9 - D_{4,9}\}$
$= 18-5$
$= 13$

$L_3 = \min\begin{Bmatrix} L_4 - D_{3,4} \\ L_5 - D_{3,5} \end{Bmatrix}$
$= \begin{matrix} 13-1 = 12 \\ 7-6 = 1 \end{matrix} \Big\} = 1$

$L_2 = \min\{L_4 - D_{2,4}\}$
$= 13-1$
$= 12$

$L_1 = \min\begin{Bmatrix} L_2 - D_{1,2} \\ L_3 - D_{1,3} \end{Bmatrix}$
$= \begin{matrix} 12-4 = 8 \\ 1-1 = 0 \end{matrix} \Big\} 0$

01/11/22

Transportation के अन्दर हो-

Vogel's Approximation method (VAM)

(Unit cost penalty method)

					Penalty	
19	30	50	10	7	(9)	19-10=9
70	30	40	60	9	(10)	40-30
40	8 (8)	70	20	18/10	(12)	20-8
5	8/0	7	14			
Penalty (21)	(22)	(10)	(10)			

(8, 18)

Step-1 → Check the lowest cost entry

Step-2 → Enter the difference b/w the lowest and second lowest cost entries and put the difference between the lowest and 2nd lowest cost entry of each row to the right of that row. Such individual differences is known as penalty

→ Check the highest penalty in row as well as column.

→ Check the min element here in which row/column the arrow is and then check the values which are in its front in term of row and column and out of them select the min one and allocate it.

→ cut the row/column which becomes 0.

5 (19)	50	10	7/2	(9)
70	40	60	9	(20)
40	70	20	10	(20)
5/0	7	14		
(21)	(10)	(10)		

min 19 = (5, 7)

50	10	2	(40)
40	60	9	(20)
70	10 (20)	10/0	(50)
7	14/4		
(10)	(10)		

min 20 = (14, 10)

50	2 (10)	2/0	(40)
40	60	9	(20)
7	4/2		
(10)	(50)		

min 0 = (2, 4)

40	2 (60)	9/7	(20)
7	2/0		
(40)	(60)		

7 (40)	7/0	(40)
7/0		
(40)		

$= 8\times8 + 5\times19 + 10\times20 + 2\times10 + 2\times60 + 7\times40$

$= 64 + 95 + 200 + 20 + 120 + 280 = 779$

02/11/22

Duality in linear programming :

Primal problem were given now will do with duality problem.

General Rules for converting any primal into its Dual :—

① First convert the objective function to maximization from if it's not.

② If a constraint has inequality sign (≥) then multiply both sides by -1 and make the inequality sign (≤)

③ If a constraint has an equally sign (=) then it is replaced by 2 constraints involving the inequalities going in opposite direction simultaneously.

For ex → An equation $x_1 + 2x_2 = 4$ is replaced by two equation that is

$$x_1 + 2x_2 = 4$$

(1) $-x_1 + 2x_2 \le 4$

$x_1 + 2x_2 \ge 4$

$-x_1 - 2x_2 \le -4$ — eq (2)

④ Every unrestricted variable is replaced by the difference of two non-negative variables

x_2 unristricted in sign → $x_1 + 2(x_2' - x_2'') \le 4$

⑤ We get the standard primal form of the given LPP in which.

(a) All the constraints have (≤) sign where the objective function is of maximization form.

(b) All the constraints have (≥) sign where the objective function is of minimization form.

⑥ Finally the Dual of the given problem is obtained by

(i) Transposing the rows and columns of constraint coefficient

(ii) Transposing the coefficient of the objective function and the right side constraints.

(iii) Changing the inequalities from (≤) to (≥) sign

(iv) Minimization of the objective function instead of maximizing it.

Vitamin	F_1	F_2	min. Daily Requirements (units)
V_1	5	7	80
V_2	6	11	~~100~~
Cost per unit	Rs. 10	Rs. 15	

→ Convert it in min

$\min Z_x = 10x_1 + 15x_2$

Sub to constraint
$$\boxed{\begin{aligned} 5x_1 + 7x_2 &\le \boxed{80} \\ 6x_1 + 11x_2 &\le \boxed{100} \end{aligned}}$$
$x_1, x_2 \ge 0$

Converting to Duality →

$\min Z_w = 80w_1 + 100w_2$

Sub ~~to~~ constraint

for x_1 → $5w_1 + 6w_2 \le 10$

for x_2 → $7w_1 + 11w_2 \le 15$

$w_1, w_2 \ge 0$

Q-1) Find the Dual of the following primal problem

$\min Z_x = 2x_2 + 5x_3$

Sub to constraint
$x_1 + x_2 \ge 2$
$2x_1 + x_2 + 6x_3 \le 6$
$x_1 - x_2 + 3x_3 = 4$

$x_1, x_2, x_3 \ge 0$

Sol ⇒ $\max Z_x = -2x_2 - 5x_3$

$= -x_1 - x_2 \le -2$

$2x_1 + x_2 + 6x_3 \le 6$

$x_1 - x_2 + 3x_3 \le 4$

$-x_1 + x_2 - 3x_3 \le -4$

$x_1 - x_2 + 3x_3 = 4$

$x_1 - x_2 + 3x_3 \le 4$ $\quad$ $x_1 - x_2 + 3x_3 \ge 4$

Converting ~~two~~ to duality:-

$\min Z_w = -2w_1 + 6w_2 + 4w_3 - 4w_4$ — Always in min

Sub to constraint
$-w_1 + 2w_2 + w_3 - w_4 \ge 0$
$-w_1 + w_2 - w_3 + w_4 \ge -2$
$(w_1) + 6w_2 + 3w_3 - 3w_4 \ge -5$

$w_1, w_2, w_3, w_4 \ge 0$

Q2 Write the dual of the following LPP

$\min Z = 3x_1 - 2x_2 + 4x_3$

Sub to constraint
$3x_1 + 5x_2 + 4x_3 \ge 7$
$6x_1 + x_2 + x_3 \ge 4$
$7x_1 - 2x_2 - x_3 \le 10$ $\quad$ $x_1, x_2, x_3 \ge 0$
$x_1 - 2x_2 + 5x_3 \ge 3$
$4x_1 + 7x_2 - 2x_3 \ge 2$

Sol ⇒ $\max Z = -3x_1 + 2x_2 - 4x_3$

$= -3x_1 - 5x_2 - 4x_3 \le -7$

$-6x_1 - x_2 - x_3 \le -4$

$7x_1 - 2x_2 - x_3 \le 10$

$-x_1 + 2x_2 - 5x_3 \le -3$

$-4x_1 - 7x_2 + 2x_3 \le -2$

= Converting to Dual.

$\min Z_w = -7w_1 - 4w_2 + 10w_3 - 3w_4 - 2w_5$

Subject to constraint

$-3w_1 - 6w_2 + 7w_3 - w_4 - 4w_5 \geq -3$

$-5w_1 - w_2 - 2w_3 + 2w_4 - 7w_5 \geq 2$

$-4w_1 - w_2 - w_3 - 5w_4 + 2w_5 \geq -4$

$w_1, w_2, w_3, w_4, w_5 \geq 0$

Q3 $\max z = 2x_1 + 3x_2 + x_3$

sub to constraint $4x_1 + 3x_2 + x_3 = 6$

$x_1 + 2x_2 + 5x_3 = 4$

and $x_1, x_2, x_3 \geq 0$

Sol⇒ $\max z = 2x_1 + 3x_2 + x_3$

sub to constraint

$4x_1 + 3x_2 + x_3 \leq 6$

$-4x_1 - 3x_2 - x_3 \leq -6$

$x_1 + 2x_2 + 5x_3 \leq 4$

$-x_1 - 2x_2 - 5x_3 \leq -4$

Converting to duality ⇒

$\min Z_w = \cancel{20}\, 6w_1 - 6w_2 + 4w_3 - 4w_4$

Sub to constraint $4w_1 - 4w_2 + w_3 - w_4 \geq 2$

$3w_1 - 3w_2 + 2w_3 - 2w_4 \geq 3$

$w_1 - w_2 + 5w_3 - 5w_4 \geq 1$

$w_1, w_2, w_3, w_4 \geq 0$

Q4 $\min z = 2x_1 + 3x_2 + 4x_3$

sub to constraint $2x_1 + 3x_2 + 5x_3 \geq 2$

$3x_1 + x_2 + 7x_3 = 3$

$x_1 + 4x_2 + 6x_3 \leq 5$

$x_1, x_2 \geq 0$ and x_3 is unrestricted

Sol⇒ Since x_3 is unrestricted $x_3 = (x'_3 - x''_3)$

• $\max Z_x = -2x_1 - 3x_2 - 4x'_3 + 4x''_3$

Sub to constraint

$-2x_1 - 3x_2 - 5x'_3 + 5x''_3 \leq -2$

$3x_1 + x_2 + 7x'_3 - 7x''_3 \leq 3$

$-3x_1 - x_2 - 7x'_3 + 7x''_3 \leq -3$

$x_1 + 4x_2 + 6x'_3 - 6x''_3 \leq 5$

Convert to duality:

$\min Z_w = -2w_1 + 3w_2 - 3w'_3 + 3w''_3 + 5w_4$

Sub to constraint

$-2w_1 + 3w_2 - 3w'_3 + 3w''_3 + w_4 \geq -2$

$-3w_1 + w_2 - w'_3 + w''_3 + 4w_4 \geq -3$

$-5w_1 + 7w_2 - 7w'_3 + 7w''_3 + 6w_4 \geq -4$

$5w_1 - 7w_2 + 7w'_3 - 7w''_3 - 6w_4 \geq 4$

$w_1, w_2, w_3, w_4 \geq 0$

$\geq \to -$ (4)

03/11/12

☆ Two Phase method :- (Artificial variable method)

In the two phase simplex method, we add artificial variables to the same constraints.
Part of simplex method

$\geq \to$ In this we will subtract 1 variable and also add one artificial variable

eg) $x_1 + x_2 \geq 3$

$x_1 + x_2 - x_3 + a_1 = 3$

Q Solve the problem $\min z = x_1 + x_2$
sub to constraints : $2x_1 + x_2 \geq 4$
$x_1 + 7x_2 \geq 7$
and $x_1, x_2 \geq 0$

(1) $\geq$ so one variable is subtracted and convert into max

$\max Z_1 = -x_1 - x_2$

Sub constraints $2x_1 + x_2 - x_3 + a_1 = 4$
$x_1 + 7x_2 + 0x_3 - x_4 + a_2 = 7$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \quad \begin{array}{cc} a_1 & a_2 \end{array}$$
$$\begin{bmatrix} 2 & 1 & -1 & 0 \\ 1 & 7 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\max Z' = -x_1 - x_2$

$\max z = 0x_1 + 0x_2 + 0x_3 + 0x_4 - 1a_1 - 1a_2$

Simplex table

B.V	C_B	X_B	x_1 (0)	x_2 (0)	x_3 (0)	x_4 (0)	A_1 (-1)	A_2 (-1)	X_B/X_2 min ratio
a_1	-1	4	2	1	-1	0	1	0	4/1 =
a_2	-1	7/2	1/2	$\boxed{7/7}$	0/2	-1/2	0/2	1/2	7/7 ←
$C_B \times X_B$		-11	-3	-8	1	1			

key element

$(-1, -1)(4, 7)$
$-4 - 7$
$= -11$

$\to x_1$ is 0 so $C_B \cdot X_1$
$= (-1, -1)(2, 1)$
$= -2 - 1$
$= -3$

$\min(\Delta_1, \Delta_2, \Delta_3, \Delta_4)$
$(-3, -8, 1, 1)$

B.V	C_B	X_B	x_1	x_2	x_3	x_4	A_1	A_2	min ratio
a_1	-1	3	$\boxed{13/7}$	0	-1	1/7	1	-1/7	3/13/7 = 21/13
a_2	-1	1	1/7	1	0	-1/7	0	1/7	7/1
		-3	-13/7	0	1	-1/7	-1	1/7	

$\min(\Delta_1\ \Delta_2\ \Delta_3\ \Delta_4)$
$(-13/7,\ 0,\ 1,\ -1/7)$

$1/7 - \frac{2}{7}$
$\frac{1 - 14}{7} = -13/7$

			0	0	0	0	-1	-1
B.V	C_B	X_B	x_1	x_2	x_3	x_4	A_1	A_2
x_1	0	21/13	1	0	-7/13	1/13	7/13	-1/13
x_2	0	10/13	0	1	1/13	-2/13		
		-31/13	0	0	6/13	1/13		

$(-x_1, -x_1)$

→ Since all $Z_j - C_j \geq 0$; $x_1 = 21/13$
$x_2 = 10/13$

$\min Z = 21/13 + 10/13 = 31/13$

Phase 1 → Take artificial variable and other 0.
Phase 2 - Take other variable and artificial variable 0.

Q.No — Use Two phase simplex method to solve the problem

$\min Z = x_1 - 2x_2 - 3x_3$

Sub to constraint
$-2x_1 + x_2 + 3x_3 = 2$
$2x_1 + 3x_2 + 4x_3 = 1$
and $x_1, x_2, x_3 \geq 0$

7/11/12

Big-M method → maximization method

Q→ Solve by using Big-M method the following LPP

$\max Z = -2x_1 - x_2$

Sub to constraint
$3x_1 + x_2 = 3$
$4x_1 + 3x_2 \geq 6$
$x_1 + 2x_2 \leq 4$
$x_1, x_2 \geq 0$.

Sol→
$3x_1 + x_2 + a_1 = 3$
$4x_1 + 3x_2 - x_3 + a_2 = 6$
$x_1 + 2x_2 + x_4 = 4$

$x_1, x_2, x_3, x_4, a_1, a_2 \geq 0$

$$\begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & A_1 & A_2 \end{array}$$
$$\begin{bmatrix} 3 & 1 & 0 & 0 & 1 & 0 \\ 4 & 3 & -1 & 0 & 0 & 1 \\ 1 & 2 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 4 \end{bmatrix}$$

			-2	-1	0	0	-m	-m	
B.V	C_B	X_B	X_1	X_2	X_3	X_4	A_1	A_2	X_B/X_1
a_1	-M	3	$\boxed{3}$	1	0	0	1	0	3/3 = 1 ←
a_2	-M	6	4	3	-1	0	0	1	6/4 = 3/2
	0	4	1	2	0	1	0	0	4/1 = 4
		-9m	2-7m	1-4m	m	0	0	0	
			↑						

$Z = C_B \times X_B$

⇒

$\Delta_1 = C_B X X_1 - C_1$

$= (-m, -m, 0)(3, 4, 1) - (-2)$

$= -3m - 4m - (-2)$

$= -7m + 2$

Key element = 3

Key element should be 1 and other 0.

$\Delta_2 = (-m, -m)(1, 3, 2) - (-1)$

$= -4m + 1$

$\Delta_3 = (-m, -m)(0, -1, 0)(0)$

$= m$

$\Delta_4 = (-m, -m)(0, 0, 1)(0)$

$= 0$

$\min(\Delta_1\ \Delta_2\ \Delta_3\ \Delta_4)$

$= (\cancel{8}-7m \quad 1-4m \quad m, 0)$

$(-5 \quad -3 \quad m \quad 0)$

B.V	C_B	X_B	X_1	X_2	X_3	X_4	A_1	A_2
X_1	-2	1	1	1/3	0	0	1/3	0
a_2	-m	6	4	3	-1	0	0	1
X_4	0	4	7	2	0	1	0	0

B.V	CB	XB	X_1	X_2	X_3	X_4	A_1	A_2	min ratio
x_1	-2	1	1	1/2	0	0	1/3	0	1/1/3 = 3
a_2	-m	2	0	$\boxed{5/3}$	1	0	4/3	1	2/5/3 = 6/5
x_4	0	3	0	5/3	0	1	-1/2	0	3/5/3 = 9/5
		2-2m	0	$(1-5m/3)$	m	0	$(2+7m)/3$	0	

B.V	CB	XB	x_1	x_2	x_3	x_4	A_1	A_2
x_1	-2	3/5	1	0	1/5	0	3/5	-1/5
x_2	-1	6/5	0	1	-3/5	0	4/5	3/5
x_3	0	1	0	0	1	1	1	-1
		-12/5	0	0	1/5	0	m-2/5	m-1/5

Max $z = -12/5$, $x_1 = 3/5$, $x_2 = 6/5$

Q. $\min z = x_1 + x_2$ → Two ~~plan~~ phase Method.

constraints $2x_1 + x_2 \geq 4$

$x_1 + 7x_2 \geq 7$

$x_1 \geq 0$

14/11/22

Q. Formulation of LPP.

The manufacturer of patent medicines is proposed to prepare a production plan for medicines A and B there are sufficient ingredient available to make 20,000 bottles of medicine A and 40,000 bottles of medicine B. But there are only 45,000 bottles into which either of the medicines can be filled further it takes 3 hrs to prepare enough material to fill 1000 bottles of medicine A and [illegible] hrs to prepare enough material to fill 1000 bottles of medicine B and there [illegible] 60 hrs available for these operation.

The profit of Rs 8 per bottle for medicine A and Rs. 7 per bottle for medicine B. Formulate these problem as a LPP.

Sol :- A → 8 Rs. profit
B → 7 Rs. profit

20,000 bottles → A
40,000 bottles → B

Total 66 hrs
3 hrs 1000 → A
1 hrs 1000 → B

20,000 , 40,000

max → $8x_1 + 7x_2$

$20x_1 + 40x_2 \le 45$

Sub to $3x_1 + x_2 \le 66$
Constraints $20x_1 + 40x_2 \le 45$

$x_1, x_2 \ge 0$
$x_1 \le 20$
$x_2 \le 40$

→ $3x_1 + x_2 + x_3 = 66$
$20x_1 + 40x_2 + x_4 = 45$

$3x_1 + x_2 + x_3 = 66$
$4x_1 + 8x_2 + x_4 = 9$

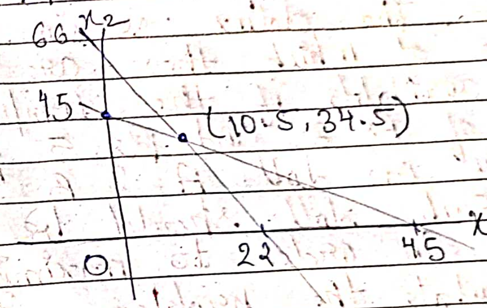
Apply the graphical method on given eq.

→ ① $3x_1 + x_2 \le 66$
let $x_1 = 0$
$\boxed{0 + x_2 = 66}$

$x_2 = 0$
$3x_1 = 66$
$\boxed{x_1 = 22}$

② $x_1 + x_2 \le 45$

$x_1 = 45$, $x_2 = 45$



$3x_1 + x_2 = 66$
$x_1 + x_2 = 45$

$2x_1 = 21$
$x_1 = 21/2 = 10.5$

$10.5 + x_2 = 45$
$x_2 = 45 - 10.5 = 34.5$

max $Z = 8x_1 + 7x_2$
$= 8(10.5) + 7(34.5)$
$= 84 + 241.5$
$= \underline{325.5}$ Ans